TWMS J. Pure Appl. Math. V.5, N.1, 2014, pp.66-79

ON INTUITIONISTIC FUZZY SOFT TOPOLOGICAL SPACES

SADI BAYRAMOV[1], CIGDEM GUNDUZ (ARAS)[2]

Abstract. In this paper, we introduce some important properties of intuitionistic fuzzy soft topological spaces and define the intuitionistic fuzzy soft closure and interior of an intuitionistic fuzzy soft set. Furthermore, intuitionistic fuzzy soft continuous mapping are given and structural characteristics are discussed and studied.

Keywords: soft set, fuzzy soft set, intuitionistic fuzzy soft topology.

AMS Subject Classification: 03E72, 08A72, 54A05, 54A40.

1. Introduction

To solve complicated problems in social sciences, economics, engineering and environment etc., we cannot use classical methods. The solutions of such problems involve the use of mathematical principles based and uncertainty and imprecision. Thus classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty. A number of theories have been proposed for dealing with uncertainties in an efficient way. Some of these are theory of fuzzy sets [25], theory of intuitionistic fuzzy sets [4], theory of vague sets, theory of interval mathematics [5,10] and theory of rough sets [13,19, 20]. However, these theories have their own difficulties. In 1999, Molodtsov [18] introduced the concept of soft set theory which is a completely new approach for modeling uncertainty. He presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Peron integration, theory of probability etc. Maji et al. [15,17] worked on soft set theory and presented an application of soft sets in decision making problems. Chen [7] introduced a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory. Some different application of soft sets were studied. The applications of soft set theory in algebraic structures was introduced by Aktas and Cagman [3]. They introduced soft groups and investigated some basic properties and compared soft sets to fuzzy and rough sets. Feng [8] gave soft semirings, soft ideals and idealistic soft semirings. Ali et al. [2] and Shabir and Irfan Ali [20] studied soft semigroups and soft ideals over a semigroup. C. Gunduz(Aras) and S. Bayramov [11,12] introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties. M.Shabir and M.Naz [22] presented soft topological sapaces. They defined some concepts of soft sets on soft topological spaces and introduced soft T_i-spaces. Since topology depends on the ideas of set theory, Tanay and Kandemir [24] introduced the concept of fuzzy

[1]Kafkas University, Kars, Turkey,
e-mail: baysadi@gmail.com
[2]Kocaeli University, Kocaeli, Turkey,
e-mail: carasgunduz@gmail.com
Manuscript received July 2013.

soft topology using the fuzzy soft sets and give the basic notions of it by following the Chang [6].

In this paper by using the concept of intuitionistic fuzzy soft set that is given in [12] we define intuitionistic fuzzy soft topological spaces. The purpose of this paper is to discuss some important properties of intuitionistic fuzzy soft topological spaces. Later we define the operations of the intuitionistic fuzzy soft closure and intuitionistic fuzzy soft interior of an intuitionistic fuzzy soft set. These operations , in fact, are a generalizations of same operations of fuzzy soft topological spaces in a broader sense. We can say that an intuitionistic fuzzy soft topological space gives a parameterized family of fuzzy bitopologies on the initial universe but the converse is not true. Finally intuitionistic fuzzy soft continuous mapping for intuitionistic fuzzy soft topological spaces are defined and some interesting results are derived which may be of value for further research.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for fuzzy soft sets.

Definition 2.1. *([18]) Let X be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X, i.e., $F : E \to P(X)$, where $P(X)$ is the power set of X.*

Definition 2.2. *([16]) Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f,A) is called a fuzzy soft set over X, where f is a mapping from A into I^X. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X.*

Definition 2.3. *([16]) For two fuzzy soft sets (f,A) and (g,B) over a common universe X, we say that (f,A) is a fuzzy soft subset of (g,B) if*

(i) $A \subset B$, and

(ii) For each $a \in A$, $f_a \leq g_a$, that is, f_a is a fuzzy subset of g_a.

This relationship is denoted by $(f,A) \widetilde{\subset} (g,B)$. Similarly, (f,A) is said to be a fuzzy soft superset of (g,B), if (g,B) is a fuzzy soft subset of (f,A). This relationship is denoted by $(f,A) \widetilde{\supset} (g,B)$.

Definition 2.4. *([16]) Two fuzzy soft sets (f,A) and (g,B) over a common universe X are said to be fuzzy soft equal if (f,A) is a fuzzy soft subset of (g,B) and (g,B) is a fuzzy soft subset of (f,A).*

Definition 2.5. *([16]) The union of two fuzzy soft sets (f,A) and (g,B) over a common universe X is the fuzzy soft set (h,C), where $C = A \cup B$ and $\forall c \in C$,*

$$h(c) = \begin{cases} f_c, \text{ if } c \in A - B \\ g(c), \text{ if } c \in B - A \\ f_c \vee g_c, \text{ if } c \in A \cap B. \end{cases}$$

This relationship is denoted by $(f,A) \widetilde{\cup} (g,B) = (h,C)$.

Definition 2.6. *([2,16]) The intersection of two fuzzy soft sets (f,A) and (g,B) over a common universe X is the fuzzy soft set (h,C), where $C = A \cap B$ and $\forall c \in C$, $h_c = f_c \wedge g_c$. This is denoted by $(f,A) \widetilde{\cap} (g,B) = (h,C)$.*

Definition 2.7. *([1]) A fuzzy soft set (f, A) over U is said to be a null fuzzy soft set and is denoted by $\widetilde{\Phi}$ if and only if for each $e \in A$, $f_e = \widetilde{0}$, where $\widetilde{0}$ is the membership function of null fuzzy set over U, which takes value 0 for all x in U.*

Definition 2.8. *([1]) A fuzzy soft set (f, A) over U is said to be an absolute fuzzy soft set and is denoted by $\widetilde{U}$ if and only if for each $e \in A$, $f_e = \widetilde{1}$, where $\widetilde{1}$ is the membership function of absolute fuzzy set over U, which takes value 1 for all x in U.*

Definition 2.9. *([16,1]) The complement of a fuzzy soft set (f, A) is the fuzzy soft set (f', A), which is denoted by $(f, A)'$ and where $f' : A \to F(U)$ is a fuzzy set-valued function i.e., for each $a \in A$, $f'(a)$ is a fuzzy set in U, whose membership function $f'_a(x) = 1 - f_a(x)$ for all $x \in U$. Here f'_a is the membership function of $f'(a)$.*

Definition 2.10. *([23]) Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if*

(1) $\Phi, \widetilde{X}$ belong to τ,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triple (X, τ, E) is called a soft topological space over X.

Let U be an initial universe, E be the set of parameters, $P(U)$ be the set of all subsets of U, $F(U)$ be the set of all fuzzy sets in U and $F\&(U; E)$ be the family of all fuzzy soft sets over U with parameters in E.

Definition 2.11. *([24]) Let (γ, X) be an element of $F\&(U; E)$, $P(\gamma, X)$ be the set of all fuzzy soft subsets of (γ, X) and $\widetilde{\tau}$ be a subfamily of $P(\gamma, X)$. Then $\widetilde{\tau}$ is called fuzzy soft topology on (γ, X) if the following conditions are satisfied:*

(i) $\widetilde{\Phi}_X, (\gamma, X) \in \widetilde{\tau}$

(ii) $(f, A), (g, B) \in \widetilde{\tau} \Rightarrow (f, A) \widetilde{\cap} (g, B) \in \widetilde{\tau}$

(iii) $\{(f, A)_k \mid k \in K\} \subset \widetilde{\tau} \Rightarrow \widetilde{\bigcup\limits_{k \in K}} (f, A)_k \in \widetilde{\tau}$.

The pair $\left(X_\gamma, \widetilde{\tau}\right)$ is called a fuzzy soft topological space. Every member of $\widetilde{\tau}$ is called $\widetilde{\tau}$-open fuzzy soft set.

Definition 2.12. *([4,5]) An intuitionistic fuzzy set A in a non-empty set X is an object having the form*

$$A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\},$$

where the functions $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $\{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

Definition 2.13. *([12]) Let $IFS(X)$ the set of all intuitionistic fuzzy sets on X and $A \subset E$. A triple (F, G, A) is called an intuitionistic fuzzy soft set over X, where F, G is a mappings from A into $IFS(X)$. That is, for each $a \in A$, $(F(a), G(a)) : X \to I$ is an intuitionistic fuzzy set on X.*

Definition 2.14. *([12]) For two intuitionistic fuzzy soft sets (F_1, G_1, A) and (F_2, G_2, B) over a common universe X, we say that (F_1, G_1, A) is an intuitionistic fuzzy soft subset of (F_2, G_2, B) and write $(F_1, G_1, A) \subset (F_2, G_2, B)$ if* *(i)* $A \subset B$,*and*

(ii) For each $a \in A$, $F_1(a) \leq F_2(a)$, $G_1(a) \geq G_2(a)$.

Definition 2.15. *([12]) Two intuitionistic fuzzy soft sets* (F_1, G_1, A) *and* (F_2, G_2, B) *over a common universe* X *are said to be equal if* $(F_1, G_1, A) \subset (F_2, G_2, B)$*and* $(F_2, G_2, B) \subset (F_1, G_1, A)$.

Definition 2.16. *([12]) Union of two intuitionistic fuzzy soft sets* (F_1, G_1, A) *and* (F_2, G_2, B) *over a common universe* X *is the intuitionistic fuzzy soft set* (H, Q, C)*, where* $C = A \cup B$ *and*

$$(H, Q)(c) = \begin{cases} (F_1(c), G_1(c)), & \text{if } c \in A - B \\ (F_2(c), G_2(c)), & \text{if } c \in B - A \\ (F_1(c) \vee F_2(c), G_1(c) \wedge G_2(c)), & \text{if } c \in B \cap A \end{cases}$$

$\forall c \in C$. *It is denoted as* $(F_1, G_1, A) \cup (F_2, G_2, B) = (H, Q, C)$.

Definition 2.17. *([2,12]) Intersection of two intuitionistic fuzzy soft sets* (F_1, G_1, A) *and* (F_2, G_2, B) *over a common universe* X *is the intuitionistic fuzzy soft set* (H, Q, C)*, where* $C = A \cap B$ *and* $(H, Q)(c) = (F_1(c) \wedge F_2(c), G_1(c) \vee G_2(c))$, $\forall c \in C$.

It is written as $(F_1, G_1, A) \cap (F_2, G_2, B) = (H, Q, C)$.

3. Intuitionistic fuzzy soft topology

In this section, we introduce some important properties of intuitionistic fuzzy soft topological spaces and define the intuitionistic fuzzy soft closure, interior of an intuitionistic fuzzy soft set.

Let X be an initial universe set and $IFS(X)$ denote the family of intuitionistic fuzzy sets on X.

Definition 3.1. *An intuitionistic fuzzy soft set* (F, G, A) *over* X *is said to be null intuitionistic fuzzy soft set and is denoted by* $\widetilde{\Phi}$*, if and only if for each* $a \in A$, $(F, G)(a) = (\underline{0}, \underline{1})$*, where* $\underline{0}$ *is the membership function of the null fuzzy set over* X *and* $\underline{1}$ *is the membership function of the absolute fuzzy set over* X.

Definition 3.2. *An intuitionistic fuzzy soft set* (F, G, A) *over* X *is said to be an absolute intuitionistic fuzzy soft set and is denoted by* $\widetilde{1}$ *if and only if for each* $a \in A$, $(F, G)(a) = (\underline{1}, \underline{0})$.

Let $IFSS(X, E)$ *be the family of all intuitionistic fuzzy soft sets over* X *with parameters in* E.

Definition 3.3. *Let* $\tau \subset IFSS(X, E)$ *be the collection of intuitionistic fuzzy soft sets over* X*, then* τ *is said to be an intuitionistic fuzzy soft topology on* X *if*

(1) $\widetilde{\Phi}, \widetilde{1}$ *belong to* τ,

(2) the union of any number of intuitionistic fuzzy soft sets in τ *belongs to* τ,

(3) the intersection of any two intuitionistic fuzzy soft sets in τ *belongs to* τ.

The triple (X, τ, E) *is called an intuitionistic fuzzy soft topological space over* X. *If* $(F, G, E) \in \tau$*, then the intuitionistic fuzzy soft sets* (F, G, E) *is said to be intuitionistic fuzzy soft open set.*

Definition 3.4. *Let* X *be an initial universe set,* E *be the set of parameters and* $\tau = \left\{\widetilde{\Phi}, \widetilde{1}\right\}$. *Then* τ *is called the intuitionistic fuzzy soft indiscrete topology on* X *and* (X, τ, E) *is said to be a intuitionistic fuzzy soft indiscrete space over* X.

Definition 3.5. *Let* X *be an initial universe set,* E *be the set of parameters and let* τ *be the collection of all intuitionistic fuzzy soft sets which can be defined over* X. *Then* τ *is called the intuitionistic fuzzy soft discrete topology on* X *and* (X, τ, E) *is said to be an intuitionistic fuzzy soft discrete space over* X.

Proposition 3.1. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X and $\tau = \{(F_s, G_s, E)\}_{se\in S}$. Then the collection $\tau_1 = \{(F_s, E)\}_{s\in S}$ and $\tau_2 = \left\{\left((G_s)', E\right)\right\}_{s\in S}$ defines a fuzzy soft topology on X.*

Proof. (1) $\widetilde{\Phi}, \widetilde{1} \in \tau$ implies that $\underline{0},\ \underline{1} \in \tau_1$ and $\underline{0},\ \underline{1} \in \tau_2$.

(2) Let $\{(F_s, G_s, E) \,|\, s \in S\}$ be a collection of intuitionistic fuzzy sets in τ. Since $(F_s, G_s, E) \in \tau$, for all $s \in S$ so that $\underset{s\in S}{\vee} (F_s, G_s, E) = \left(\underset{s\in S}{\vee} F\ _s,\ \underset{s\in S}{\wedge} G\ _s\right) \in \tau$, thus $\underset{s\in S}{\vee} (F_s, E) \in \tau_1$. Similarly, $\underset{s\in S}{\vee} (G'_s, E) \in \tau_2$ is proven.

(3) Let $(F_1, G_1, E), (F_2, G_2, E) \in \tau$. Since$(F_1, G_1, E) \wedge (F_2, G_2, E) = (F_1 \wedge F_2, G_1 \vee G_2, E) \in \tau$ so $(F_1, E) \wedge (F_2, E) \in \tau_1$ and $(G'_1, E) \wedge (G'_2, E) \in \tau_2$. Thus $\tau_1,\ \tau_2$ defines a fuzzy soft topology on X. These topologies are not equal. □

Now we show that the converse of above proposition is not true.

Example 3.1. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFSS(X)$ on X, for $1 \le i \le 5$, are defined as follows

$$
\begin{array}{l}
(F_1, G_1)(e_1)(x_1) = \left(\frac{1}{3}, \frac{1}{2}\right),\ (F_1, G_1)(e_1)(x_2) = \left(\frac{1}{4}, \frac{1}{3}\right) \\
(F_1, G_1)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right),\ (F_1, G_1)(e_2)(x_2) = \left(\frac{1}{2}, 0\right), \\
(F_2, G_2)(e_1)(x_1) = \left(\frac{1}{2}, \frac{1}{3}\right),\ (F_2, G_2)(e_1)(x_2) = \left(\frac{1}{3}, \frac{1}{2}\right), \\
(F_2, G_2)(e_2)(x_1) = \left(\frac{1}{5}, \frac{2}{5}\right),\ (F_2, G_2)(e_2)(x_2) = \left(\frac{2}{5}, \frac{1}{2}\right), \\
(F_3, G_3)(e_1)(x_1) = \left(\frac{1}{2}, \frac{1}{3}\right),\ (F_3, G_3)(e_1)(x_2) = \left(\frac{1}{3}, \frac{1}{3}\right), \\
(F_3, G_3)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right),\ (F_3, G_3)(e_2)(x_2) = \left(\frac{1}{2}, 0\right), \\
(F_4, G_4)(e_1)(x_1) = \left(\frac{1}{3}, \frac{1}{2}\right),\ (F_4, G_4)(e_1)(x_2) = \left(\frac{1}{4}, \frac{1}{2}\right), \\
(F_4, G_4)(e_2)(x_1) = \left(\frac{1}{5}, \frac{2}{5}\right),\ (F_4, G_4)(e_2)(x_2) = \left(\frac{2}{5}, \frac{1}{2}\right), \\
(F_5, G_5)(e_1)(x_1) = \left(\frac{1}{2}, \frac{1}{2}\right),\ (F_5, G_5)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{2}\right), \\
(F_5, G_5)(e_2)(x_1) = \left(\frac{1}{2}, \frac{9}{20}\right),\ (F_5, G_5)(e_2)(x_2) = \left(\frac{1}{2}, \frac{1}{2}\right).
\end{array}
$$

Then

$$\tau_1 = \{\underline{0},\ \underline{1},\ (F_1, E),\ (F_2, E),\ (F_3, E),\ (F_4, E),\ (F_5, E)\}$$

and

$$\tau_2 = \{\underline{0},\ \underline{1},\ (G_1, E),\ (G_2, E),\ (G_3, E),\ (G_4, E),\ (G_5, E)\}$$

are fuzzy soft topologies on X and $F_i(e_j)(x_k) + G_i(e_j)(x_k) \le 1$.

Here $\tau = \left\{\widetilde{\Phi}, \widetilde{1}, (F_1, G_1, E), (F_2, G_2, E), (F_3, G_3, E), (F_4, G_4, E), (F_5, G_5, E)\right\}$ is not an intuitionistic fuzzy soft topology on X, because $(F_1, G_1, E) \wedge (F_5, G_5, E) = (F, G, E)$,

$$
\begin{array}{l}
(F, G)(e_1)(x_1) = \left(\frac{1}{3}, \frac{1}{2}\right),\ (F, G)(e_1)(x_2) = \left(\frac{1}{4}, \frac{1}{2}\right), \\
(F, G)(e_2)(x_1) = \left(\frac{1}{2}, \frac{9}{20}\right),\ (F, G)(e_2)(x_2) = \left(\frac{1}{2}, \frac{1}{2}\right)
\end{array}
$$

and so $(F_1, G_1, E) \wedge (F_5, G_5, E) = (F, G, E) \notin \tau$.

Proposition 3.2. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X. Then the collection $\tau_{1e} = \{F(e) \,|\, (F, G, E) \in \tau\}$ $\tau_{2e} = \{G'(e) \,|\, (F, G, E) \in \tau\}$for each $e \in E$, defines two fuzzy topologies on X.*

Proof. From proposition 3.6 $\tau_1 = \{(F_s, E)\}_{s\in S}$ and $\tau_2 = \left\{\left((G_s)', E\right)\right\}_{s\in S}$ are fuzzy soft topologies . For each $e \in E, \tau_{1e} = \{F(e) \,|\, (F, G, E) \in \tau\}$and $\tau_{2e} = \{G'(e) \,|\, (F, G, E) \in \tau\}$are fuzzy topologies on X. These two fuzzy topologies τ_{1e}, τ_{2e}will denote as τ_e and will call as fuzzy bitopology on X.

Proposition 3.8 shows that corresponding to each parameter $e \in E$, we have a fuzzy bitopology τ_e on X. Thus an intuitionistic fuzzy soft topology on X gives a parameterized family of fuzzy bitopologies on X. □

Now we show that the converse of above proposition is not true.

Example 3.2. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFS(X)$ on X, for $1 \leq i \leq 5$, are defined as follows

$$\begin{array}{l} (F_1,G_1)(e_1)(x_1) = \left(\frac{1}{2},\frac{1}{2}\right),\ (F_1,G_1)(e_1)(x_2) = \left(\frac{2}{3},\frac{1}{3}\right), \\ (F_1,G_1)(e_2)(x_1) = \left(\frac{1}{5},\frac{4}{5}\right),\ (F_1,G_1)(e_2)(x_2) = \left(\frac{3}{5},\frac{2}{5}\right), \\ (F_2,G_2)(e_1)(x_1) = \left(\frac{1}{3},\frac{2}{3}\right),\ (F_2,G_2)(e_1)(x_2) = (1,0), \\ (F_2,G_2)(e_2)(x_1) = \left(\frac{1}{7},\frac{6}{7}\right),\ (F_2,G_2)(e_2)(x_2) = \left(\frac{4}{5},\frac{1}{5}\right), \\ (F_4,G_4)(e_1)(x_1) = \left(\frac{1}{2},\frac{1}{2}\right),\ (F_4,G_4)(e_1)(x_2) = (1,0), \\ (F_4,G_4)(e_2)(x_1) = \left(\frac{1}{5},\frac{4}{5}\right),\ (F_4,G_4)(e_2)(x_2) = \left(\frac{4}{5},\frac{1}{5}\right), \\ (F_5,G_5)(e_1)(x_1) = \left(\frac{2}{3},\frac{1}{3}\right),\ (F_5,G_5)(e_1)(x_2) = (1,0), \\ (F_5,G_5)(e_2)(x_1) = \left(\frac{1}{10},\frac{9}{10}\right),\ (F_5,G_5)(e_2)(x_2) = \left(\frac{2}{5},\frac{3}{5}\right). \end{array}$$

Then

$$\tau_{e_1} = \{(\underline{0},\ \underline{1}),(\ \underline{1},\underline{0}),(F_1(e_1),G_1(e_1)),\ldots,(F_5(e_1),G_5(e_1))\}$$

and

$$\tau_{e_2} = \{(\underline{0},\ \underline{1}),(\ \underline{1},\underline{0}),(F_1(e_2),G_1(e_2)),\ldots,(F_5(e_2),G_5(e_2))\}$$

are fuzzy bitopologies on X. Here $\tau = \left\{\widetilde{\Phi}, \widetilde{1}, (F_1,G_1,E), \ldots, (F_5,G_5,E)\right\}$ is not an intuitionistic fuzzy soft topology on X, because $(F_3,G_3,E) \vee (F_5,G_5,E) = (F,G,E)$,

$$\begin{array}{l} (F,G)(e_1)(x_1) = \left(\frac{2}{3},\frac{1}{3}\right),\ (F,G)(e_1)(x_2) = (1,0), \\ (F,G)(e_2)(x_1) = \left(\frac{1}{7},\frac{6}{7}\right),\ (F,G)(e_2)(x_2) = \left(\frac{3}{5},\frac{2}{5}\right) \end{array}$$

and so $(F_3,G_3,E) \vee (F_5,G_5,E) = (F,G,E) \notin \tau$.

Proposition 3.3. *Let (X,τ_1,E) and (X,τ_2,E) be two intuitionistic fuzzy soft topological spaces over the same universe X, then $(X,\tau_1 \wedge \tau_2,E)$ is an intuitionistic fuzzy soft topological spaces over X.*

Remark. The union of two intuitionistic fuzzy soft topologies on X may not be an intuitionistic fuzzy soft topology on X.

Definition 3.6. *The complement of an intuitionistic fuzzy soft set (F,G,E) is denoted by $(F,G,E)'$ and is defined by $(F,G,E)' = (G,F,E)$.*

Proposition 3.4. *(De-Morgan's laws) Let $\{(F_\alpha,G_\alpha,E)\}_\alpha$ the family of intuitionistic fuzzy soft sets over X, then*

a) $\left\{\bigvee\limits_\alpha (F_\alpha,G_\alpha,E)\right\}' = \bigwedge\limits_\alpha (F_\alpha,G_\alpha,E)'$,

b) $\left\{\bigwedge\limits_\alpha (F_\alpha,G_\alpha,E)\right\}' = \bigvee\limits_\alpha (F_\alpha,G_\alpha,E)'$.

Definition 3.7. *Let (X,τ,E) be an intuitionistic fuzzy soft topological space over X. An intuitionistic fuzzy soft set (F,G,E) is said to be intuitionistic fuzzy soft closed set in X, if its complement $(F,G,E)'$ belongs to τ.*

Proposition 3.5. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X. Then*

(1) $\widetilde{\Phi}, \widetilde{1}$ are intuitionistic fuzzy soft closed sets over X,

(2) For each $(F_s, G_s, E)' \in \tau$, $\left(\underset{s\in S}{\wedge}(F_s, G_s, E)\right)' \in \tau$,

(3) For each $(F_1, G_1, E)', (F_2, G_2, E)' \in \tau$, $((F_1, G_1, E) \vee (F_2, G_2, E))' \in \tau$.

Proof. The proof of the theorem is obtained from the definition of intuitionistic fuzzy soft topological spaces and De-Morgan's laws for intuitionistic fuzzy soft sets. □

Definition 3.8. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X and (F, G, E) be an intuitionistic fuzzy soft set over X. The intuitionistic fuzzy soft closure of (F, G, E), denoted by $\overline{(F, G, E)}$ is the intersection of all intuitionistic fuzzy soft closed super sets of (F, G, E).*

Clearly $\overline{(F, G, E)}$ is the smallest intuitionistic fuzzy soft closed set over X which contains (F, G, E).

Theorem 3.1. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X, (F_1, G_1, E) and (F_2, G_2, E) are intuitionistic fuzzy soft sets over X. Then*

(1) $\overline{\widetilde{\Phi}} = \widetilde{\Phi}$ and $\overline{\widetilde{1}} = \widetilde{1}$,

(2) $(F_1, G_1, E) \subset \overline{(F_1, G_1, E)}$

(3) (F_1, G_1, E) is an intuitionistic fuzzy soft closed set if and only if $(F_1, G_1, E) = \overline{(F_1, G_1, E)}$,

(4) $\overline{(F_1, G_1, E)} = \overline{\overline{(F_1, G_1, E)}}$,

(5) $(F_1, G_1, E) \subset (F_2, G_2, E)$ implies $\overline{(F_1, G_1, E)} \subset \overline{(F_2, G_2, E)}$,

(6) $\overline{(F_1, G_1, E) \vee (F_2, G_2, E)} = \overline{(F_1, G_1, E)} \vee \overline{(F_2, G_2, E)}$.

Proof. (1) and (2) are obvious.

(3) If (F_1, G_1, E) is an intuitionistic fuzzy soft closed set over X then (F_1, G_1, E) is itself an intuitionistic fuzzy soft closed set over X which contains (F_1, G_1, E). So (F_1, G_1, E) is the smallest intuitionistic fuzzy soft closed set containing (F_1, G_1, E) and $(F_1, G_1, E) = \overline{(F_1, G_1, E)}$.

(4) Since $\overline{(F_1, G_1, E)}$ is an intuitionistic fuzzy soft closed set therefore by part (3) we have $\overline{(F_1, G_1, E)} = \overline{\overline{(F_1, G_1, E)}}$.

(5) Suppose that $(F_1, G_1, E) \subset (F_2, G_2, E)$. Then every intuitionistic fuzzy soft closed super set of (F_2, G_2, E) will also contain (F_1, G_1, E). This means every intuitionistic fuzzy soft closed super set of (F_2, G_2, E) is also an intuitionistic fuzzy soft closed super set of (F_1, G_1, E). Hence the intersection of intuitionistic fuzzy soft closed super sets of (F_1, G_1, E) is contained in the intersection of intuitionistic fuzzy soft closed super sets of (F_2, G_2, E). Thus $\overline{(F_1, G_1, E)} \subset \overline{(F_2, G_2, E)}$.

(6) Since $(F_1, G_1, E) \subset (F_1, G_1, E) \vee (F_2, G_2, E)$ and $(F_2, G_2, E) \subset (F_1, G_1, E) \vee (F_2, G_2, E)$, from by part (5),

$$\overline{(F_1, G_1, E)} \subset \overline{(F_1, G_1, E) \vee (F_2, G_2, E)}, \overline{(F_2, G_2, E)} \subset \overline{(F_1, G_1, E) \vee (F_2, G_2, E)}.$$

Hence $\overline{(F_1, G_1, E)} \vee \overline{(F_2, G_2, E)} \subset \overline{(F_1, G_1, E) \vee (F_2, G_2, E)}$.

Conversely suppose that $(F_1, G_1, E) \subset \overline{(F_1, G_1, E)}$ and $(F_2, G_2, E) \subset \overline{(F_2, G_2, E)}$. So (F_1, G_1, E). From proposition 3.15, $\overline{(F_1, G_1, E)} \vee \overline{(F_2, G_2, E)}$ is an intuitionistic fuzzy soft closed set over X being the union of two intuitionistic fuzzy soft closed sets. Then $\overline{(F_1, G_1, E) \vee (F_2, G_2, E)} \subset \overline{(F_1, G_1, E)} \vee \overline{(F_2, G_2, E)}$. Then $\overline{(F_1, G_1, E) \vee (F_2, G_2, E)} = \overline{(F_1, G_1, E)} \vee \overline{(F_2, G_2, E)}$ is obtained. □

Definition 3.9. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X and (F, G, E) be an intuitionistic fuzzy soft set over X. Then the associated closure of (F, G, E) is an intuitionistic fuzzy soft set over X, denoted by $\left(\overline{F}, \overline{G}, E\right)$ and defined as closure of $(F(e), G(e))$*

in fuzzy bitopological space (X, τ_e). *For each* $e \in E$ $\left(\overline{F}, \overline{G}\right)(e) = \left(\underset{s}{\wedge} G_s(e), \underset{s}{\vee} F_s(e)\right)$, *where* $(F_s, G_s, E) \in \tau$ *and* $(F(e), G(e)) \leq (F_s(e), G_s(e))$.

Proposition 3.6. *Let* (X, τ, E) *be an intuitionistic fuzzy soft topological space over* X *and* (F, G, E) *be an intuitionistic fuzzy soft set over* X. *Then* $(\overline{F}, \overline{G}, E) \subset \overline{(F, G, E)}$.

Proof. For any $e \in E$, $\left(\overline{F(e)}, \overline{G(e)}\right)$ is the smallest closed set in (X, τ_e) which contains $(F(e), G(e))$. So $\overline{(F, G, E)} = (H, Q, E)$ then $(H, Q)(e)$ is also a closed set in (X, τ_e) containing $(F(e), G(e))$. This implies that $\left(\overline{F(e)}, \overline{G(e)}\right) \leq (H, Q)(e)$. Thus $(\overline{F}, \overline{G}, E) \subset \overline{(F, G, E)}$. □

Example 3.3. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFS(X)$ on X, for $1 \leq i \leq 5$, are defined as follows

$$\begin{array}{l}
(F_1, G_1)(e_1)(x_1) = \left(\frac{1}{5}, \frac{1}{4}\right), \ (F_1, G_1)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{3}\right), \\
(F_1, G_1)(e_2)(x_1) = \left(\frac{1}{4}, \frac{2}{3}\right), \ (F_1, G_1)(e_2)(x_2) = (1, 0), \\
(F_2, G_2)(e_1)(x_1) = \left(\frac{1}{3}, \frac{2}{3}\right), \ (F_2, G_2)(e_1)(x_2) = \left(\frac{1}{5}, \frac{3}{5}\right), \\
(F_2, G_2)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right), \ (F_2, G_2)(e_2)(x_2) = \left(\frac{1}{4}, \frac{2}{3}\right), \\
(F_3, G_3)(e_1)(x_1) = \left(\frac{1}{3}, \frac{1}{4}\right), \ (F_3, G_3)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{3}\right), \\
(F_3, G_3)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right), \ (F_3, G_3)(e_2)(x_2) = (1, 0), \\
(F_4, G_4)(e_1)(x_1) = \left(\frac{1}{5}, \frac{2}{3}\right), \ (F_4, G_4)(e_1)(x_2) = \left(\frac{1}{5}, \frac{3}{5}\right), \\
(F_4, G_4)(e_2)(x_1) = \left(\frac{1}{4}, \frac{2}{3}\right), \ (F_4, G_4)(e_2)(x_2) = \left(\frac{1}{4}, \frac{2}{3}\right), \\
(F_5, G_5)(e_1)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right), \ (F_5, G_5)(e_1)(x_2) = \left(\frac{3}{5}, \frac{1}{5}\right), \\
(F_5, G_5)(e_2)(x_1) = \left(\frac{2}{3}, \frac{1}{4}\right), \ (F_5, G_5)(e_2)(x_2) = (1, 0).
\end{array}$$

Then (X, τ, E) is an intuitionistic fuzzy soft topological space over X. If (F, G, E) is defined as follows

$(F, G)(e_1)(x_1) = \left(\frac{1}{4}, \frac{2}{3}\right)$, $(F, G)(e_1)(x_2) = \left(\frac{1}{5}, \frac{2}{5}\right)$,$(F, G)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{3}\right)$, $(F_1, G_1)(e_2)(x_2) = \left(\frac{1}{3}, \frac{1}{2}\right)$. Then $\overline{(F, G, E)} = (G_4, F_4, E)$,$\left(\overline{F}, \overline{G}\right)(e_1) = ((G_2 \wedge G_4)(e_1), (F_2 \vee F_4)(e_1))$, $\left(\overline{F}, \overline{G}\right)(e_2) = (G_4(e_2), F_4(e_2))$. Thus $\overline{(F, G, E)} \neq \left(\overline{F}, \overline{G}, E\right)$.

Definition 3.10. *Let* (X, τ, E) *be an intuitionistic fuzzy soft topological space over* X *and* (F, G, E) *be an intuitionistic fuzzy soft set over* X. *Define the interior of* (F, G, E) *as the join of all the intuitionistic fuzzy soft open subsets contained in* (F, G, E) *and denote it by* $(F, G, E)^\circ$.

Theorem 3.2. *Let* (X, τ, E) *be an intuitionistic fuzzy soft topological space over* X *and* (F_1, G_1, E) *and* (F_2, G_2, E) *are intuitionistic fuzzy soft sets over* X. *Then*

(1) $\widetilde{\Phi}^\circ = \widetilde{\Phi}$ *and* $\widetilde{1}^\circ = \widetilde{1}$,

(2) $(F_1, G_1, E)^\circ \subset (F_1, G_1, E)$

(3) (F_1, G_1, E) *is an intuitionistic fuzzy soft open set if and only if* $(F_1, G_1, E)^\circ = (F_1, G_1, E)$,

(4) $((F_1, G_1, E)^\circ)^\circ = (F_1, G_1, E)^\circ$,

(5) $(F_1, G_1, E) \subset (F_2, G_2, E)$ *implies* $(F_1, G_1, E)^\circ \subset (F_2, G_2, E)^\circ$,

(6) $((F_1, G_1, E) \wedge (F_2, G_2, E))^\circ = (F_1, G_1, E)^\circ \wedge (F_2, G_2, E)^\circ$.

Definition 3.11. *Let* (X, τ, E) *be an intuitionistic fuzzy soft topological space over* X *and* (F, G, E) *be an intuitionistic fuzzy soft set over* X. *Then the associated interior of* (F, G, E) *is an intuitionistic fuzzy soft set over* X, *denoted by* (F°, G°, E) *and defined as* $(F^\circ, G^\circ)(e) = \left(\underset{s}{\vee} F_s(e), \underset{s}{\wedge} G_s(e)\right)$, *where* $(F_s, G_s, E) \in \tau$ *and* $(F_s(e), G_s(e)) \leq (F(e), G(e))$.

Proposition 3.7. *Let* (X, τ, E) *be an intuitionistic fuzzy soft topological space over* X *and* (F, G, E) *be an intuitionistic fuzzy soft set over* X. *Then* $(F, G, E)^\circ \subset (F^\circ, G^\circ, E)$.

Proof. For each $e \in E$, $(F^\circ, G^\circ)(e)$is the biggest open fuzzy set in (X, τ_e)which belonging $(F, G)(e)$.Since $(F, G, E)^\circ \subset (F, G, E)$, then $(F, G, E)^\circ \subset (F^\circ, G^\circ, E)$ is satisfied. $\square$

Example 3.4. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. The intuitionistic fuzzy soft sets $(F_i, G_i) : E \to IFS(X)$ on X, for $1 \leq i \leq 5$, are defined as follows

$$\begin{array}{l}
(F_1, G_1)(e_1)(x_1) = \left(\frac{1}{5}, \frac{1}{4}\right),\ (F_1, G_1)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{3}\right), \\
(F_1, G_1)(e_2)(x_1) = \left(\frac{1}{4}, \frac{2}{3}\right),\ (F_1, G_1)(e_2)(x_2) = (1, 0), \\
(F_2, G_2)(e_1)(x_1) = \left(\frac{1}{3}, \frac{2}{3}\right),\ (F_2, G_2)(e_1)(x_2) = \left(\frac{1}{5}, \frac{3}{5}\right), \\
(F_2, G_2)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right),\ (F_2, G_2)(e_2)(x_2) = \left(\frac{1}{4}, \frac{2}{3}\right), \\
(F_3, G_3)(e_1)(x_1) = \left(\frac{1}{3}, \frac{1}{4}\right),\ (F_3, G_3)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{3}\right), \\
(F_3, G_3)(e_2)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right),\ (F_3, G_3)(e_2)(x_2) = (1, 0), \\
(F_4, G_4)(e_1)(x_1) = \left(\frac{1}{5}, \frac{2}{3}\right),\ (F_4, G_4)(e_1)(x_2) = \left(\frac{1}{5}, \frac{3}{5}\right), \\
(F_4, G_4)(e_2)(x_1) = \left(\frac{1}{4}, \frac{2}{3}\right),\ (F_4, G_4)(e_2)(x_2) = \left(\frac{1}{4}, \frac{2}{3}\right), \\
(F_5, G_5)(e_1)(x_1) = \left(\frac{1}{2}, \frac{1}{4}\right),\ (F_5, G_5)(e_1)(x_2) = \left(\frac{3}{5}, \frac{1}{5}\right), \\
(F_5, G_5)(e_2)(x_1) = \left(\frac{2}{3}, \frac{1}{4}\right),\ (F_5, G_5)(e_2)(x_2) = (1, 0).
\end{array}$$

Then (X, τ, E) is an intuitionistic fuzzy soft topological space over X. If (F, G, E) is defined as follows

$$\begin{aligned}
(F, G)(e_1)(x_1) &= \left(\frac{1}{2}, \frac{1}{5}\right),\ (F, G)(e_1)(x_2) = \left(\frac{1}{2}, \frac{1}{5}\right),\ (F, G)(e_2)(x_1) = \left(\frac{2}{3}, \frac{1}{3}\right), \\
(F_1, G_1)(e_2)(x_2) &= (1, 0)
\end{aligned}$$

Then

$$(F, G, E)^\circ = (F_1, G_1, E) \vee (F_4, G_4, E) = (F_1, G_1, E),$$

$$(F^\circ, G^\circ)(e_1) = \left(\overset{4}{\underset{i=1}{\vee}} F_i(e_1), \overset{4}{\underset{i=1}{\wedge}} G_i(e_1)\right), \quad (F^\circ, G^\circ)(e_2) = (F_1(e_2) \vee F_4(e_2), G_1(e_2) \wedge G_4(e_2)).$$

Thus $(F, G, E)^\circ \neq (F^\circ, G^\circ, E)$.

Proposition 3.8. *Let (X, τ, E) be an intuitionistic fuzzy soft topological space over X and (F, G, E) be an intuitionistic fuzzy soft set over X. Then $(F, G, E)^\circ = \left(\overline{(F, G, E)'}\right)'$.*

Proof. Let (F, G, E) be an intuitionistic fuzzy soft set. We show that $((F, E)^\circ)' = \left(\overline{(F, E)'}\right)$. From the definition of interior of (F, G, E), we have

$$((F, G, E)^\circ)' = \left(\underset{\substack{(H,Q,E) \subset (F,G,E) \\ (H,Q,E) \in \tau}}{\vee} (H, Q, E)\right)' = \underset{\substack{(H,Q,E) \subset (F,G,E) \\ (H,Q,E) \in \tau}}{\wedge} (H, Q, E)' = \left(\overline{(F, G, E)'}\right)$$

Hence $(F, G, E)^\circ = \left(\overline{(F, G, E)'}\right)'$ is obtained. $\square$

4. Intuitionistic fuzzy soft continuous mappings

Definition 4.1. *Let (X, τ, E), (Y, τ', E) be two intuitionistic fuzzy soft topological spaces, $f : X \to Y$ be a mapping and (F, G, E) be an intuitionistic fuzzy soft set over X. Then the image of (F, G, E) under the mapping f, denoted by $f((F, G, E)) = (f(F), f(G), E)$, is an intuitionistic fuzzy soft set over Y defined by*

$$(f(F), f(G))(e)(y) = \left(\underset{f(x)=y}{\vee} F(e)(x), \underset{f(x)=y}{\wedge} G(e)(x)\right) \text{for each } e \in E.$$

Definition 4.2. *Let (X,τ,E), (Y,τ',E)be two intuitionistic fuzzy soft topological spaces, $f: X\to Y$ be a mapping and (F,G,E) be an intuitionistic fuzzy soft set over Y. Then the pre-image of (F,G,E) under the mapping f, denoted by $f^{-1}(F,G,E)=\left(f^{-1}(F),f^{-1}(G),E\right)$, is an intuitionistic fuzzy soft set over X defined by $\left(f^{-1}(F),f^{-1}(G)\right)(e)(x)=(F(e)(f(x)),G(e)(f(x)))$ for each $e\in E$.*

Proposition 4.1. *Let (F_1,G_1,E), (F_2,G_2,E) be two intuitionistic fuzzy soft sets over X and Y, respectively, and $f: X\to Y$ be a mapping. Then*
(1) $(F_1,G_1,E)\subset f^{-1}(f(F_1,G_1,E))$,
(2) $f\left(f^{-1}(F_1,G_1,E)\right)\subset(F_1,G_1,E)$.

Proposition 4.2. *Let $\{(F_i,G_i,E)\}_{i\in I}$ be a family of intuitionistic fuzzy soft sets over Y. Then*
(1) $f^{-1}\left(\underset{i\in I}{\vee}(F_i,G_i,E)\right)=\underset{i\in I}{\vee}f^{-1}(F_i,G_i,E)$
(2) $f^{-1}\left(\underset{i\in I}{\wedge}(F_i,G_i,E)\right)=\underset{i\in I}{\wedge}f^{-1}(F_i,G_i,E)$.

Definition 4.3. *Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. For each $(F,G,E)\in\tau'$, if $f^{-1}(F,G,E)\in\tau$, then $f:(X,\tau,E)\to(Y,\tau',E)$ is said to be intuitionistic fuzzy soft continuous mapping of intuitionistic fuzzy soft topological spaces.*

Theorem 4.1. *Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. Then the following conditions are equivalent: (1) $f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping, (2) For each intuitionistic fuzzy soft closed (H,Q,E) over Y, $f^{-1}((H,Q,E))$ is an intuitionistic fuzzy soft closed set over X, (3) For each intuitionistic fuzzy soft set (F,G,E) over X, $f\left(\overline{(F,G,E)}\right)\subset\overline{(f(F,G,E))}$, (4) For each intuitionistic fuzzy soft set (H,Q,E) over Y, $\overline{(f^{-1}(H,Q,E))}\subset f^{-1}\left(\overline{(H,Q,E)}\right)$, (5) For each intuitionistic fuzzy soft set (H,Q,E) over Y, $f^{-1}((H,Q,E)^{\circ})\subset\left(f^{-1}(H,Q,E)\right)^{\circ}$.*

Proof. (1)⇒(2) Let (H,Q,E) be an intuitionistic fuzzy soft closed set over Y. Then $(H,Q,E)'\in\tau'$. By part (1), we have $f^{-1}\left((H,Q,E)'\right)\in\tau$. Since

$$f^{-1}\left((H,Q,E)'\right)=\left(f^{-1}(H,Q,E)\right)'\in\tau,$$

$f^{-1}((H,Q,E))$ is an intuitionistic fuzzy soft closed set over X. (2)⇒(3) Let (F,G,E) be an intuitionistic fuzzy soft set over X. Since

$$(F,G,E)\subset f^{-1}(f(F,G,E)),\ f(F,G,E)\subset f\overline{(F,G,E)},$$

we have

$$(F,G,E)\subset f^{-1}(f(F,G,E))\subset f^{-1}\left(f\overline{(F,G,E)}\right).$$

By part (2), since $f^{-1}\overline{(f(F,G,E))}$ is an intuitionistic fuzzy soft closed set over X, $\overline{(F,G,E)}\subset f^{-1}\overline{(f(F,G,E))}$. Thus $f\left(\overline{(F,G,E)}\right)\subset f\left(f^{-1}\overline{(f(F,G,E))}\right)\subset\overline{f(F,G,E)}$ is obtained.

(3)⇒(4) Let (H,Q,E) be an intuitionistic fuzzy soft set over Y and $f^{-1}(H,Q,E)=(F,G,E)$. By part (3), we have

$$f\left(\overline{(F,G,E)}\right)=f\left(\overline{f^{-1}(H,Q,E)}\right)\subset\overline{f(f^{-1}(H,Q,E))}\subset\overline{(H,Q,E)}.$$

Then $\overline{(f^{-1}(H,Q,E))}=\overline{(F,G,E)}\subset f^{-1}\left(f\overline{(F,G,E)}\right)\subset f^{-1}\left(\overline{(H,Q,E)}\right)$.

(4)⇒(5) Let (H,Q,E) be a soft set over Y. Substituting $(H,Q,E)'$for condition in (4). Then $\overline{f^{-1}\left((H,Q,E)'\right)} \subset f^{-1}\left(\overline{(H,Q,E)'}\right)$. From proposition 3.26, since $(H,Q,E)^{\circ} = \left(\overline{(H,Q,E)'}\right)'$, then we have

$$f^{-1}\left((H,Q,E)^{\circ}\right) = f^{-1}\left(\left(\overline{(H,Q,E)'}\right)'\right) = \left(f^{-1}\left(\overline{(H,Q,E)'}\right)\right)' \subset$$
$$\left(\overline{f^{-1}\left((H,Q,E)'\right)}\right)' = \left(f^{-1}(H,Q,E)\right)^{\circ}.$$

(5)⇒(1) Let (H,Q,E)be an intuitionistic fuzzy soft open set over Y. Then since

$$\left(f^{-1}(H,Q,E)\right)^{\circ} \subset f^{-1}(H,Q,E) = f^{-1}\left((H,Q,E)^{\circ}\right) \subset \left(f^{-1}(H,Q,E)\right)^{\circ},$$

$\left(f^{-1}(H,Q,E)\right)^{\circ} = f^{-1}(H,Q,E)$is obtained. This implies that $f^{-1}(H,Q,E)$ is an intuitionistic fuzzy soft open set over X. □

Example 4.1. Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. If τ' is the intuitionistic fuzzy soft indiscrete topology on Y, then $f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping.

Example 4.2. Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. If τ is the intuitionistic fuzzy soft discrete topology on X, then $f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping.

Proposition 4.3. *If mapping $f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping, then for each $e\in E$,$f:(X,\tau_e)\to(Y,\tau'_e)$ is a fuzzy continuous mapping of bitopological spaces.*

Proof. Let $(U,V)\in\tau'_{\alpha}$. Then there exists an intuitionistic fuzzy soft open set (F,G,E) over Y such that $(F(e),G(e))=(U,V)$. Since $f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping, $f^{-1}(F,G,E)$ is an intuitionistic fuzzy soft open set over X and $f^{-1}(F,G)(e) = f^{-1}(F(e),G(e)) = \left(f^{-1}(U), f^{-1}(V)\right)$ is a fuzzy open sets. This implies that f is a fuzzy continuous mapping. □

Example 4.3. Let $X=\{x_1,x_2,x_3\}$, $Y=\{y_1,y_2,y_3\}$ and $E=\{e_1,e_2\}$.
Here $\{(F_i,G_i,E):\ 1\le i\le 10\}$ are intuitionistic fuzzy soft sets over X and (H_1,Q_1,E), (H_2,Q_2,E) are intuitionistic fuzzy soft sets over Y, defined as follows:

$$\begin{aligned}
&(F_1,G_1)(e_1)(x_1)=\left(\tfrac{2}{3},\tfrac{1}{3}\right),\ (F_1,G_1)(e_1)(x_2)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ (F_1,G_1)(e_1)(x_2)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\\
&(F_1,G_1)(e_2)(x_1)=\left(\tfrac{5}{7},\tfrac{2}{7}\right),\ (F_1,G_1)(e_2)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),(F_1,G_1)(e_2)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\\
&(F_2,G_2)(e_1)(x_1)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_2,G_2)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_2,G_2)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\\
&(F_2,G_2)(e_2)(x_1)=(0,1),\ (F_2,G_2)(e_2)(x_2)=(1,0),(F_2,G_2)(e_2)(x_2)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\\
&(F_3,G_3)(e_1)(x_1)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ (F_3,G_3)(e_1)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\ (F_3,G_3)(e_1)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\\
&(F_3,G_3)(e_2)(x_1)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ (F_3,G_3)(e_2)(x_2)=(0,1),(F_3,G_3)(e_2)(x_2)=(0,1),\\
&(F_4,G_4)(e_1)(x_1)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_4,G_4)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_4,G_4)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\\
&(F_4,G_4)(e_2)(x_1)=(0,1),\ (F_4,G_4)(e_2)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),(F_4,G_4)(e_2)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\\
&(F_5,G_5)(e_1)(x_1)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_5,G_5)(e_1)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\ (F_5,G_5)(e_1)(x_2)=\left(\tfrac{1}{5},\tfrac{4}{5}\right),\\
&(F_5,G_5)(e_2)(x_1)=(0,1),\ (F_5,G_5)(e_2)(x_2)=(0,1),(F_5,G_5)(e_2)(x_2)=(0,1),\\
&(F_6,G_6)(e_1)(x_1)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ (F_6,G_6)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\ (F_6,G_6)(e_1)(x_2)=\left(\tfrac{1}{4},\tfrac{3}{4}\right),\\
&(F_6,G_6)(e_2)(x_1)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ (F_6,G_6)(e_2)(x_2)=(1,0),(F_6,G_6)(e_2)(x_2)=\left(\tfrac{1}{2},\tfrac{1}{2}\right),
\end{aligned}$$

$$\begin{array}{l}
(F_7,G_7)(e_1)(x_1)=\left(\frac{2}{3},\frac{1}{3}\right),\ (F_7,G_7)(e_1)(x_2)=\left(\frac{1}{2},\frac{1}{2}\right),\ (F_7,G_7)(e_1)(x_2)=\left(\frac{1}{2},\frac{1}{2}\right),\\
(F_7,G_7)(e_2)(x_1)=\left(\frac{5}{7},\frac{2}{7}\right),\ (F_7,G_7)(e_2)(x_2)=(1,0),(F_7,G_7)(e_2)(x_2)=\left(\frac{1}{2},\frac{1}{2}\right),\\
(F_8,G_8)(e_1)(x_1)=\left(\frac{1}{4},\frac{3}{4}\right),\ (F_8,G_8)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\ (F_8,G_8)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\\
(F_8,G_8)(e_2)(x_1)=(0,1),\ (F_8,G_8)(e_2)(x_2)=(0,1),(F_8,G_8)(e_2)(x_2)=(0,1),\\
(F_9,G_9)(e_1)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ (F_9,G_9)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\ (F_9,G_9)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\\
(F_9,G_9)(e_2)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ (F_9,G_9)(e_2)(x_2)=(0,1),(F_9,G_9)(e_2)(x_2)=(0,1),\\
(F_{10},G_{10})(e_1)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ (F_{10},G_{10})(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\ (F_{10},G_{10})(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\\
(F_{10},G_{10})(e_2)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ (F_{10},G_{10})(e_2)(x_2)=\left(\frac{1}{5},\frac{1}{5}\right),(F_{10},G_{10})(e_2)(x_2)=\left(\frac{1}{5},\frac{1}{5}\right).\\
(H_1,Q_1)(e_1)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ (H_1,Q_1)(e_1)(x_2)=\left(\frac{2}{3},\frac{1}{3}\right),\ (H_1,Q_1)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\\
(H_1,Q_1)(e_2)(x_1)=\left(\frac{1}{5},\frac{4}{5}\right),\ (H_1,Q_1)(e_2)(x_2)=\left(\frac{5}{7},\frac{2}{7}\right),(H_1,Q_1)(e_2)(x_2)=(1,0),\\
(H_2,Q_2)(e_1)(x_1)=\left(\frac{1}{4},\frac{3}{4}\right),\ (H_2,Q_2)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\ (H_2,Q_2)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\\
(H_2,Q_2)(e_2)(x_1)=(0,1),\ (H_2,Q_2)(e_2)(x_2)=\left(\frac{1}{2},\frac{1}{2}\right),(H_2,Q_2)(e_2)(x_2)=\left(\frac{1}{3},\frac{2}{3}\right).
\end{array}$$

If we get the mapping $f : X \to Y$ defined as

$$f : X \to Y,\ \ f(x_1) = y_2\,,\ f(x_2) = f(x_3) = y_1,$$

then f is not an intuitionistic fuzzy soft continuous mapping. Since

$$\begin{array}{l}
f^{-1}(H_2,Q_2)(e_1)(x_1)=\left(\frac{1}{4},\frac{3}{4}\right),\ f^{-1}(H_2,Q_2)(e_1)(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),\ f^{-1}(H_2,Q_2)(e_1)(x_3)=\left(\frac{1}{4},\frac{3}{4}\right),\\
f^{-1}(H_2,Q_2)(e_2)(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),\ f^{-1}(H_2,Q_2)(e_2)(x_2)=(0,1),\ f^{-1}(H_2,Q_2)(e_2)(x_3)=(0,1)
\end{array}$$

we have $f^{-1}(H_2, Q_2, E) \notin \tau$. Also,

$$\begin{array}{l}
\tau_{e_1}=\left\{\left(\underline{0},\underline{1}\right),\left(\underline{1},\underline{0}\right),(F_1,G_1)(e_1),(F_2,G_2)(e_1),(F_3,G_3)(e_1),(F_5,G_5)(e_1),(F_6,G_6)(e_1)\right\}\\
\tau_{e_2}=\left\{\begin{array}{l}(F_1,G_1)(e_2),(F_2,G_2)(e_2),(F_3,G_3)(e_2),(F_4,G_4)(e_2),(F_6,G_6)(e_2),(F_7,G_7)(e_2),\\(F_{10},G_{10})(e_2)\end{array}\right\}
\end{array}$$

$$\begin{array}{l}
\tau'_{e_1}=\left\{\left(\underline{0},\underline{1}\right),\left(\underline{1},\underline{0}\right),(H_1,Q_1)(e_1),(H_2,Q_2)(e_1)\right\}\\
\tau'_{e_2}=\left\{\left(\underline{0},\underline{1}\right),\left(\underline{1},\underline{0}\right),(H_1,Q_1)(e_2),(H_2,Q_2)(e_2)\right\}.
\end{array}$$

The mapping $f : (X, \tau_{e_1}) \to (Y, \tau'_{e_1})$ is a fuzzy continuous mapping, because

$$\begin{array}{l}
f^{-1}(H_1,Q_1)(e_1))(x_1)=\left(\frac{2}{3},\frac{1}{3}\right),\ f^{-1}(H_1,Q_1)(e_1))(x_2)=\left(\frac{1}{2},\frac{1}{2}\right),f^{-1}(H_1,Q_1)(e_1))(x_3)=\left(\frac{1}{2},\frac{1}{2}\right),\\
f^{-1}(H_2,Q_2)(e_1))(x_1)=\left(\frac{1}{4},\frac{3}{4}\right),f^{-1}(H_2,Q_2)(e_1))(x_2)=\left(\frac{1}{4},\frac{3}{4}\right),f^{-1}(H_2,Q_2)(e_1))(x_3)=\left(\frac{1}{4},\frac{3}{4}\right)
\end{array}$$

Hence, $f^{-1}(H_1, Q_1)(e_1) = (F_1, G_1)(e_1)$, $f^{-1}(H_2, Q_2)(e_1) = (F_2, G_2)(e_1)$ is obtained. Similarly, the mapping $f : (X, \tau_{e_2}) \to (Y, \tau'_{e_2})$ is a fuzzy continuous mapping, because

$$\begin{array}{l}
f^{-1}(H_1,Q_1)(e_2))(x_1)=\left(\frac{5}{7},\frac{2}{7}\right),\ f^{-1}(H_1,Q_1)(e_2))(x_2)=\left(\frac{1}{5},\frac{4}{5}\right),f^{-1}(H_1,Q_1)(e_1))(x_3)=\left(\frac{1}{5},\frac{4}{5}\right),\\
f^{-1}(H_2,Q_2)(e_2))(x_1)=\left(\frac{1}{2},\frac{1}{2}\right),f^{-1}(H_2,Q_2)(e_2))(x_2)=(0,1),f^{-1}(H_2,Q_2)(e_2))(x_3)=(0,1)
\end{array}$$

Thus $f^{-1}(H_1, Q_1)(e_2) = (F_1, G_1)(e_2)$, $f^{-1}(H_2, Q_2)(e_2) = (F_3, G_3)(e_2)$ is obtained.

5. Conclusions

The purpose of this paper is to discuss some important properties of intuitionistic fuzzy soft topological spaces and define the intuitionistic fuzzy soft closure of an intuitionistic fuzzy soft set. Later, we provide the intuitionistic fuzzy soft interior of an intuitionistic fuzzy soft set and investigate some of its basic properties. We can say that an intuitionistic fuzzy soft topological space gives a parameterized family of fuzzy bitopologies on the initial universe but the converse is not true. Finally, intuitionistic fuzzy soft continuous mappings for intuitionistic fuzzy soft topological spaces are defined and some interesting results are obtained, which may be of value for further research.

6. Acknowledgement

The authors are very much grateful to referees and editor for their valuable comments and suggestions that helped in improving this paper.

References

[1] Ahmad, B., Kharal, A., (2009), On Fuzzy Soft Sets, Adv. Fuzzy Syst.

[2] Ali, M.I., Feng, F., Liu, X.Y., Min, W.K., Shabir, M., (2009) On some new operations in soft set theory, Comput. Math. Appl., 57, pp.1547-1553.

[3] Aktas, H., N. Cagman, N., (2007), Soft sets and soft group, Information Science, 177, pp.2726-2735.

[4] Atanassov, K., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, pp.87-96.

[5] Atanassov, K., (1994), Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 64, pp.159-174.

[6] Chang, C.L., (1968), Fuzzy topological spaces, J. Math. Anal. Appl., 24, pp.182-190.

[7] Chen, D., (2005), The parametrization reduction of soft sets and its applications, Comput. Math. Appl., 49, pp.757-763.

[8] Feng, F., Jun, Y.B., Zhao, X., (2008), Soft semirings, Comput. Math. Appl., 56, pp.2621-2628.

[9] Ge, X., Li, Z., Ge, Y., (2011), Topological spaces and soft sets, Journal of Computational Analysis and Applications, 13, pp.881-885.

[10] Gorzalzany, M.B. (1987), A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems, 21, pp.1-17.

[11] Gunduz(Aras), C., Bayramov, S., (2011), Fuzzy soft modules, International Math. Forum, 6(11), pp.517-527.

[12] Gunduz(Aras), C., Bayramov, S., (2011), Intuitionistic fuzzy soft modules, Computers and Mathematics with Application, 62, pp.2480-2486.

[13] Irfan Ali, M., (2011), A note on soft sets, rough sets and fuzzy soft sets, Applied Soft Computing, 11, pp.3329-3332.

[14] Irfan Ali, M., Davvaz, B., Shabir, M., (2012), Generalized fuzzy S-acts and their characterization by soft s-acts, Neural Computing and Application, 21, (Suppl 1):S9–S17.

[15] Maji, P.K., Bismas, R., Roy A.R., (2003), Soft set theory, Comput. Math. Appl., 45, pp.555-562.

[16] Maji, P.K., Bismas, R., Roy A.R., (2001), Fuzzy soft sets, Journal of Fuzzy Mathematics, 9(3), pp.589-602.

[17] Maji, P.K., Roy, A.R., Bismas, R., (2002), An Application of soft sets in a decision making problem, Comput. Math. Appl., 44, pp.1077-1083.

[18] Molodtsov, D., (1999), Soft set theory- first results, Comput. Math. Appl., 37, pp.19-31.

[19] Muhammad, Shabir, Muhammad, Irfan Ali, Tanzeela, Shaheen, (2013), Another approach to soft rough sets, Knowledge-Based Systems, Volume 40, pp.72-80.

[20] Pawlak, Z., (1982), Rough sets, Int.J.Comput.Sci., 11, pp.341-356.

[21] Qiu-Mei Sun, Zi-Liong Zhang, Jing Liu, (2008), Soft sets and soft modules, Lecture Notes in Comput. Sci., 5009, pp.403-409.

[22] Shabir, M., Irfan Ali, M., (2009), Soft ideals and generalized fuzzy ideals in semigroups, New Math. Nat. Comput., 5, pp.599-615.

[23] Shabir, M., Naz, M., (2011), On soft topological spaces, Comput. Math. Appl., 61, pp.1786-1799.

[24] Tanay, B., Burç Kandemir, M., (2011), Topological structure of fuzzy soft sets, Comput. Math. Appl., 61, pp.2952-2957.

[25] Zadeh, L.A., (1965), Fuzzy sets, Inf. Control, 8, pp.338-353.

Sadi Bayramov was born in 1953. He recieved his Ph.D. degree in Baku State University in 1985. He worked at the Baku State University between the years 1975-1996. His area of interests are topology, fuzzy topology, fuzzy algebra and their applications. Since 1996 he is with the Kafkaz University, Turkey.

Cigdem Gunduz Aras was born in 1972. In 1994 she graduated from the Mathematics Department of the Faculty of Science in Blacksea Technical University. In 1997 she has completed her master's degree in Kocaeli University. In 2002 she has finished her doctoral thesis in the same university. Her area of interests are topology, fuzzy topology, fuzzy algebra and their applications.